Signals & Systems

Lecture# 1

CLASSIFICATION OF SIGNALS

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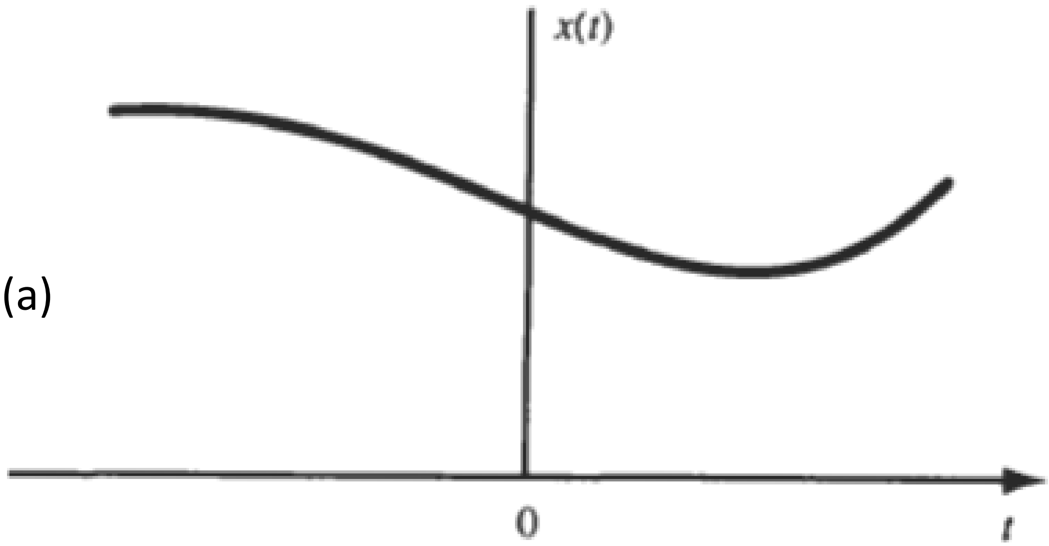
Mathematically, a signal is represented as a function of an independent variable t.
Usually t represents time.
Thus, a signal is denoted by x(t).

1. Continuous-Time and Discrete-Time Signals:

A signal x(t) is a continuous-time signal if **t is a continuous variable**. If **t is a discrete variable**, that is, x(t) is defined at discrete times, then x(t) is a discrete-time signal.
Since a discrete-time signal is defined at discrete times, a discrete time signal is often identified as a sequence of numbers, denoted by **{x} or x[n], where n = integer.**

(a)



(b)

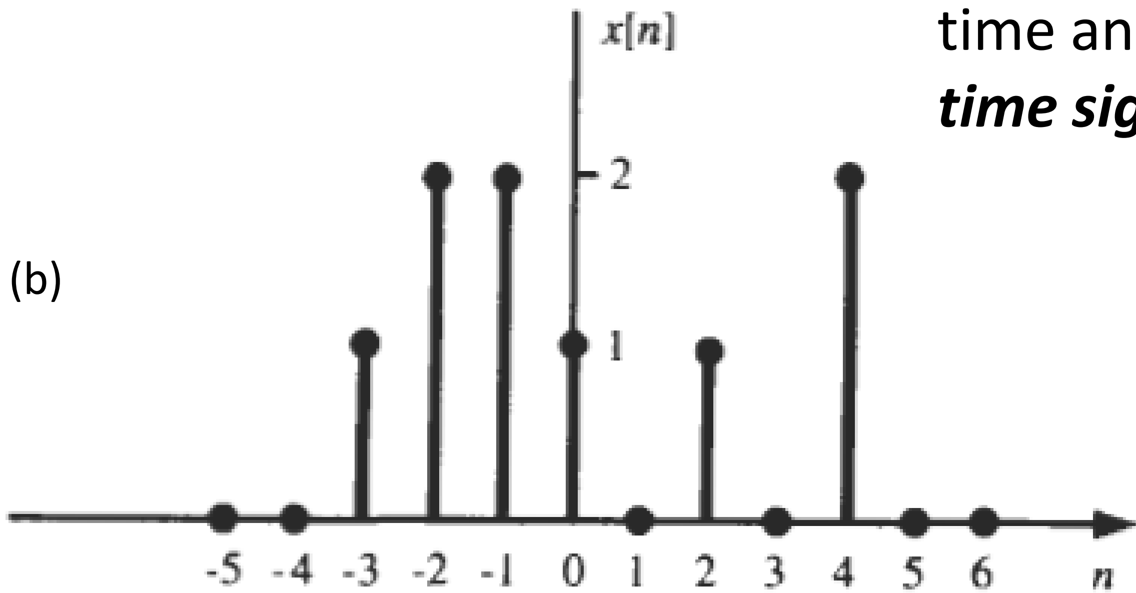


Fig.1(a): continuous-time and ***(b) discrete-time signals.***

A discrete-time signal x[n] may represent a phenomenon for which the independent variable is inherently discrete.

A discrete-time signal x[n] may be obtained by sampling a continuous-time signal x(t) such as

$$x(t_0),\ x(t_1), \ldots, x(t_n), \ldots$$

or in a shorter form as

$$x[0], x[1], \ldots, x[n], \ldots$$

$$\text{or} \qquad x_0, x_1, \ldots, x_n, \ldots$$

where we understand that

$$x_n = x[n] = x(t_n)$$

and x_n's are called samples and the time interval between them is called the sampling interval. When the sampling intervals are equal (uniform sampling), then $x_n = x[n] = x(nT_s)$

2. Analog and Digital Signals:

If a continuous-time signal x(t) can take on any value in the continuous interval (a, b), where a may be - ∞ *and b may be + ∞, then the continuous-time signal x(t) is called an* analog signal. If a discrete-time signal x[n] can take on only a finite number of distinct values, then we call this signal a digital signal.

3. Real and Complex Signals:

A signal x(t) is a real signal if its value is a real number, and a signal x(t) is a complex signal if its value is a complex number. A general complex signal x(t) is a function of the form

$$x(t) = x_1(t) + jx_2(t)$$

where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$.

4. Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time t .

Random signals are those signals that take random values at any given time and must be characterized statistically.

5. Even and Odd Signals:

A signal $x(t)$ or $x[n]$ is referred to as an even signal if

$x(-t) = x(t)$

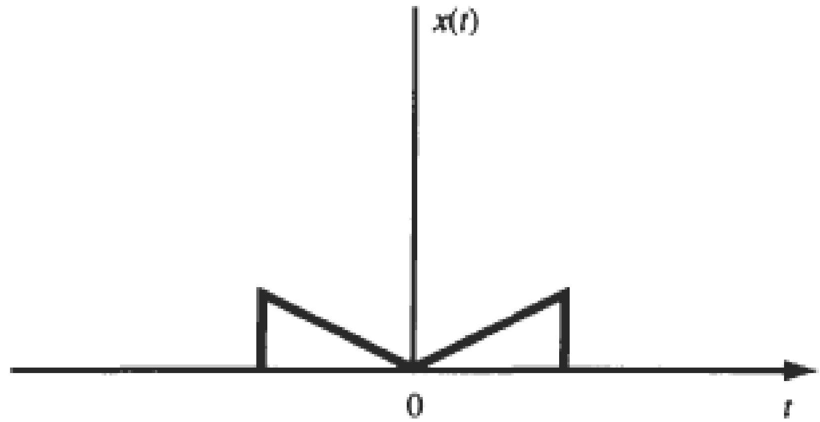
$x[-n] = x[n]$

A signal $x(t)$ or $x[n]$ is referred to as an odd signal if

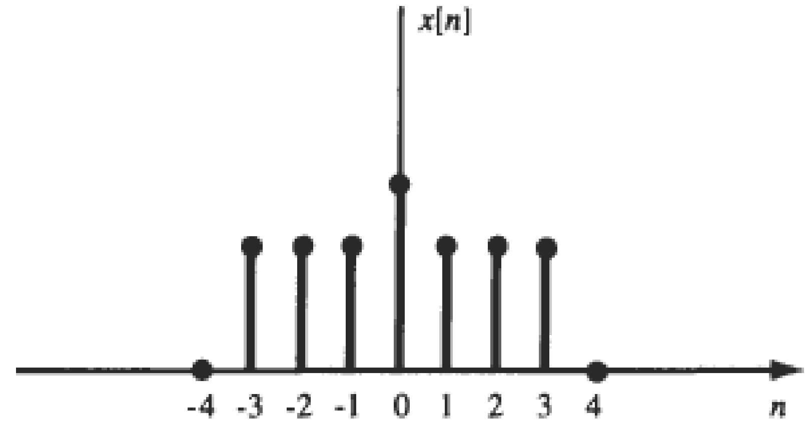
$x(-t) = -x(t)$

$x[-n] = -x[n]$

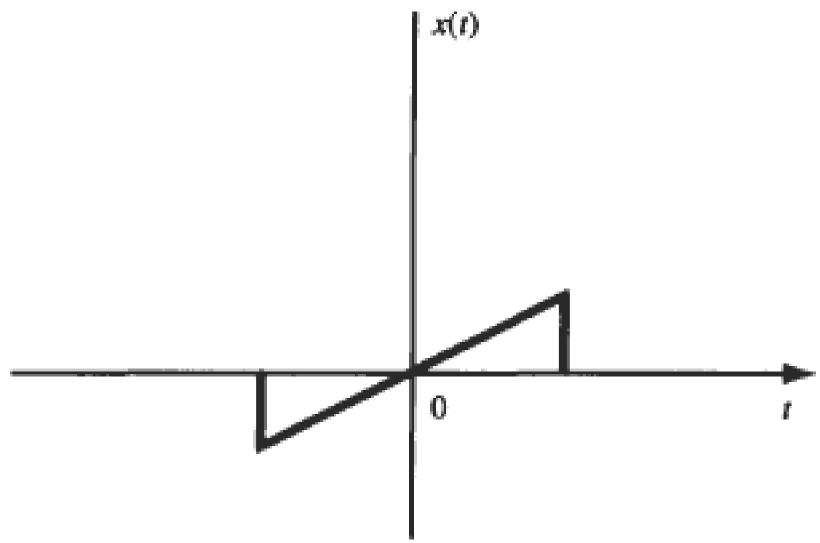
Examples:



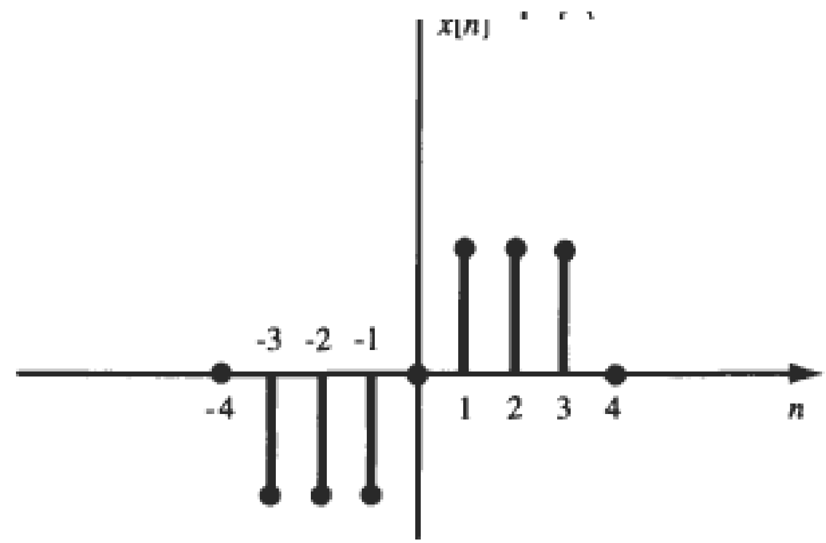
(a)



(b)



(c)



(d)

Any signal x(t) or x[n] can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

$$x(t) = x_e(t) + x_o(t)$$

$$x[n] = x_e[n] + x_o[n]$$

where $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$ even part of $x(t)$

$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$ even part of $x[n]$

$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$ odd part of $x(t)$

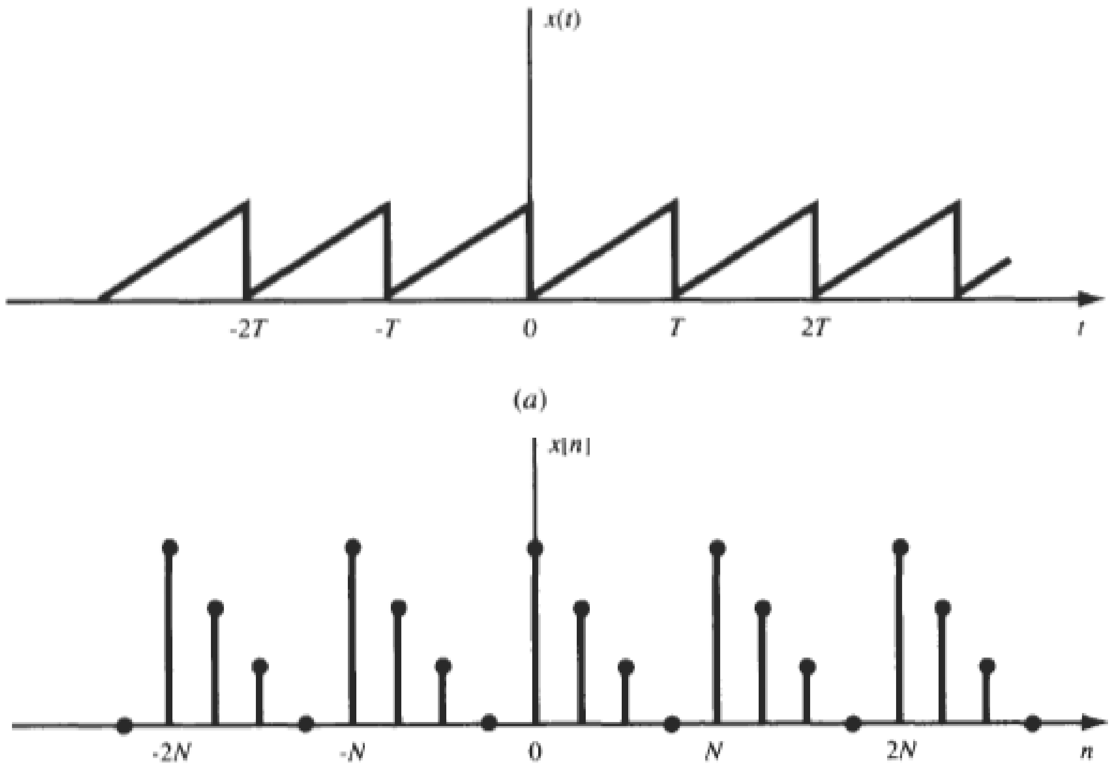
$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$ odd part of $x[n]$

6. Periodic and Nonperiodic Signals:

A continuous-time signal x (t) is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \qquad \text{all } t$$

An example of such a signal is given in Fig.



$$x(t + mT) = x(t)$$

for all t and any integer m. The fundamental period T_0 of $x(t)$ is the smallest positive value of T

7. Energy and Power Signals:

For an arbitrary continuous-time signal $x(t)$, the *normalized energy content* E of $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

The *normalized average power* P of $x(t)$ is defined as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Similarly, for a discrete-time signal $x[n]$, the normalized energy content E of $x[n]$ is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

The normalized average power P of $x[n]$ is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Based on definitions the following classes of signals are defined:

1. $x(t)$ (or $x[n]$) is said to be an *energy* signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. $x(t)$ (or $x[n]$) is said to be a *power* signal (or sequence) if and only if $0 < P < \infty$, thus implying that $E = \infty$.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.